

Indian Statistical Institute
B. Math. III Year
Mid-Semestral Examination 2008-2009
Topology

Date: 19-09-2008

Total Marks:35

Instructor: J. Biswas

Answer all questions.

1. Give an example of a topological space which is
 - a) Hausdorff but not regular
 - b) limit point compact but not compact
 - c) metrizable but not second countable
 - d) second countable but not metrizable
 - e) path connected but not locally connected.

Please justify each answer.

[20]

2. Let $H = \prod_{n \in \mathbb{Z}_+} [0, 1/n]$ denote the Hilbert cube. Prove that the product topology and the uniform topology on H , inherited as a subspace of \mathbb{R}^ω , are the same. [5]
3. Prove that a topological space is second countable and normal, if and only if, it can be imbedded as a subspace of the Hilbert cube (with the uniform topology). [5]
4. Define an equivalence relation on the points of the n -sphere, S^n , as follows: for $x, y \in S^n$ we have $x \sim y$, if and only if, either $y = x$ or $y = -x$. Let $p : S^n \rightarrow S^n / \sim$ denote the quotient map. Here S^n has the usual subspace topology inherited from the euclidean topology of \mathbb{R}^{n+1} , and S^n / \sim has the quotient topology. Is p an open map? Is p a closed map? [5]